Gravitational Potential Energy

① $E_g = -\frac{GMm}{r}$

$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(1)}{(6.38\times10^{8})}$

$E_g = \boxed{-625\ 181.8\ \text{J}}$

② $E_g = -\frac{GMm}{r}$

$= -\frac{(6.67\times10^{-11})(5.98\times10^{24})(7.34\times10^{22})}{(3.8\times10^{8})}$

$E_g = \boxed{-7.7\times10^{28}\ \text{J}}$

③ $\Delta E_g = E_g' - E_g$

$= \left(-\frac{GMm}{r'}\right) - \left(-\frac{GMm}{r}\right)$

$= -GMm\left(\frac{1}{r'} - \frac{1}{r}\right)$

$= -(6.67\times10^{-11})(5.98\times10^{24})(1)\left[\frac{1}{1.276\times10^{7}} - \frac{1}{6.38\times10^{6}}\right]$

$\Delta E_g = \boxed{31\ 259\ 091\ \text{J}}$

④ $\Delta E_g = -GMm\left(\frac{1}{r'} - \frac{1}{r}\right)$

$= -(6.67\times10^{-11})(5.98\times10^{24})(5)\left[\frac{1}{7975000} - \frac{1}{6.38\times10^{6}}\right]$

$\Delta E_g = \boxed{62\ 518\ 182\ J}$

⑤ $\Delta E_K = -\Delta E_g$

$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\left[-GMm\left(\frac{1}{r'} - \frac{1}{r}\right)\right]$

$\frac{1}{2}\cancel{m}v_f^2 = GM\cancel{m}\left(\frac{1}{r'} - \frac{1}{r}\right)$

$\frac{1}{2}v_f^2 = (6.67\times10^{-11})(7.34\times10^{22})\left[\frac{1}{1.74\times10^{6}} - \frac{1}{1827000}\right]$

$\frac{1}{2}v_f^2 = 133984.127$

$v_f = \boxed{517.7\ m/s}$

(6) $\Delta E_k = -\Delta E_g$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = -\left[-GMm\left(\frac{1}{r'} - \frac{1}{r}\right)\right]$$

$$-\frac{1}{2}\cancel{m}v_i^2 = GM\cancel{m}\left(\frac{1}{r'} - \frac{1}{r}\right)$$

$$-\frac{1}{2}v_i^2 = (6.67\times10^{-11})(5.98\times10^{24})\left[\frac{1}{12760000} - \frac{1}{6.38\times10^6}\right]$$

$$-\frac{1}{2}v_i^2 = -31259090.91$$

$$v_i = \boxed{7906.8 \text{ m/s}}$$

(7) $\Delta E_g = -GMm\left(\frac{1}{r'} - \frac{1}{r}\right)$

$$= -(6.67\times10^{-11})(5.98\times10^{24})(1)\left[\frac{1}{6.48\times10^6} - \frac{1}{6.38\times10^6}\right]$$

$$\Delta E_g = \boxed{964\,786.7565 \text{ J}}$$

Using $E_g = mgh$

$$= (1)(9.8)(100\,000)$$

$$\Delta E_g = 980\,000 \text{ J}$$

$$\% \text{ error} = \left(1 - \frac{964\,786.8}{980\,000}\right) \times 100\% = \boxed{1.6\%}$$

8) a) $\Delta E_g = -GMm\left(\frac{1}{r'} - \frac{1}{r}\right)$

$= -(6.67\times10^{-11})(1.98\times10^{30})(5.98\times10^{24})\left[\frac{1}{1.52\times10^{11}} - \frac{1}{1.47\times10^{11}}\right]$

$\Delta E_g = \boxed{1.77\times10^{32}\ \text{J}}$

b) Moving fastest at closest approach because E_g is at minimum. $\therefore$ E_K is at maximum.

$\therefore$ $\boxed{\text{Perihelion}}$

$\text{Max } \Delta E_K = \text{Max } \Delta E_g = \boxed{1.77\times10^{32}\ \text{J}}$